SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) Prove that the sum of the probabilities of any event A and its complement $\overline{A}$ is one.

(b) Show that the probability of the impossible event is zero.

17. (a) Let $\{A_n\}$, $n = 1, 2, \ldots\ldots$ be a non decreasing sequence of events and let A be their alternative, prove that $P(A) = \lim_{n \to \infty} P(A_n)$.

(b) If event A and B satisfy the condition $A \subset B$, then prove that $P(A) \leq P(B)$.

18. Prove that the expected value of the product of an arbitrary finite number of independent random variables, whose expected values exist, equals the product of the expected values of these variables.

19. Prove that the distribution on function $F(x, y)$ of a two-dimensional random variables (X, Y) is uniquely determined by the class of all one-dimensional distribution functions of $tx + uy$, where t and a run over all possible real values.

20. State and prove Borel-Cantelli Lemma.

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NOVEMBER/DECEMBER 2024

23PMA32 — PROBABILITY THEORY

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Random event.
2. Explain difference of event.
3. Define independent event.
4. What is distribution function?
5. State the expected value of random variable.
6. Define characteristic function.
7. What do you mean by one-point distribution?
8. Explain stochastic convergence.
9. Define limit distribution function.
10. State strong law of large numbers.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that the sum of the probability of any event A and its complement $\overline{A}$ is one.

Or

(b) Explain Marginal Distribution.

12. (a) Show that A random variable X is of the continuous type with the density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}.$$

Or

(b) Prove that if a random variable Y can take on only non-negative values and has expected value $E(Y)$, then for an arbitrary positive number K, $P(Y \geq K) \leq \frac{E(Y)}{K}$.

13. (a) Prove that the characteristic function of the sum of the arbitrary finite number of independent random variables equals the product of their characteristic functions.

Or

(b) Show that the random variable X has a poisson distribution, that is

$$P_K = e^{-\lambda} \frac{\lambda^K}{K!} \quad (K = 0, 1, 2, \ldots\ldots\ldots).$$

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14. (a) Prove that let the random variable X_n have a binomial distribution defined by the formula.

$$P(X_n = r) = \frac{n!}{r!(n-r)!} P^r (1-P)^{n-r}, \text{ where}$$

r takes on the values 0, 1, 2, ... n. If for $n = 1, 2, \ldots\ldots$ the relation $P = \frac{\lambda}{n}$ holds, where $\lambda > 0$ is a constant, then $\lim_{n\to\infty} P(X_n = r) = \frac{\lambda^n}{r!} e^{-\lambda}$.

Or

(b) Explain Gamma distribution.

15. (a) Prove that the sequence of random variables $\{X_n\}$ given by equations is stochastically convergent to 0, that is for any $\Sigma > 0$ we have $\lim_{n\to\infty} P(|X_n| > \epsilon) = 0$.

Or

(b) Show that let $\{X_k\}$ $(k = 1, 2, \ldots\ldots)$ be a sequence of independent random variables with the same distribution and with expected value $E(X_k) = m$. Then the sequence $\{Y_n\}$, where $Y_n = \frac{X_1 + X_2 + \ldots + X_n}{n}$, is stochastically convergent to m.